Classification of large N superconformal gauge theories

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Based on [Agarwal, JS 1912.12881][Agarwal, K.H. Lee, JS 2007.16165] [Maruyoshi, Nardoni, JS work in progress]

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Motivation & Introduction

AdS/CFT correspondence

- It is an exact equivalence of non-gravitational system and a gravitational system.
- Gravity as a "hologram" of non-gravitational system.
- It gives a precise quantitative description of quantum gravity in anti-de Sitter space.
- **Black hole** in gravity = **Thermal state** in field ${ \bullet }$ theory

[Maldacena '97] [Gubser-Klebanov-Polyakov '98][Witten '98]

Conformal field theory (CFT) in d-dimension = quantum gravity in (d+1)-dim anti-de Sitter (AdS) space.







The most familiar cases of AdS/ CFT are 'derived' from string/Mtheory.

Probe large number of D/M-branes and take the near-horizon limit.









Familiar cases of AdS/CFT - Multiple D/M-branes probing singularities

- D3-brane probing Calabi-Yau 3-fold singularity (8 Q's) 4d $\mathcal{N}=1$ quiver gauge theory \leftrightarrow Type IIB on AdS₅×SE⁵
- N M2-branes probing $\mathbb{C}^4/\mathbb{Z}_k$ (24 Q's) 3d $\mathcal{N}=6$ U(N)_k×U(N)_{-k} theory \leftrightarrow M-theory on AdS₄×S⁷/ \mathbb{Z}_k
- N M5-branes probing flat space (32 Q's) 6d $\mathcal{N}=(2, 0) A_{N-1} SCFT \leftrightarrow M$ -theory on AdS₇×S⁴
- D1-D5 brane system with M₄ transverse direction (8 Q's) 2d $\mathcal{N}=(4, 4)$ SCFT \leftrightarrow Type IIB on AdS₃×S³×M₄ (T⁴ or K3)



But, AdS/CFT correspondence goes beyond these well-controlled cases.

Any Conformal Field Theory in d-dim A "Quantum Gravity" in AdS in (d+1)-dim

CFT can be thought of as a definition of a certain 'quantum gravity' in AdS.

Q) What sort of 'quantum gravity' for a given CFT?

The space of AdS QGs = CFTs





Gravity with no UV completion "AdS Swampland" = Inconsistent (or part of a) CFT

> "Exotic" Gravity = Generic consistent CFT

"Einstein-like" Gravity = "Holographic" CFT

Holographic CFT When do we have a semi-classical gravity dual?

- Existence of the Large N limit: We need a family of theories parametrized by N.
- Locality in the bulk: Large Gap in the (single-trace) higher-spin operators
- Phase structure: Sparse spectrum at low energy <=> Hawking-Page phase transition
- Correlators of the low-lying operators factorize

[Heemskerk, Penedones, Polchinski, Sully] [El-Showk, Papadodimas], many others...

Large N gauge theories

- These conditions seem to be satisfied for 'any' large N gauge theories in the 't Hooft limit.
- Sparse spectrum from the gauge invariant operators with $\Delta_{\rm gap} = \mathcal{O}(1)$
- Confinement/deconfinement transition
 <=> Hawking-Page transition

Is this *always* true? Does large *N* gauge theory always admit a semi-classical gravity dual?



Classifying large N theories

- Let us classify all possible supersymmetric large N gauge theories in 4d with the following conditions:
 - The gauge group is simple: G=SU(N), SO(N), Sp(N).
 - The flavor symmetry is fixed as we take large N limit.
 - No superpotential (at the moment)
- The second condition is important from the AdS dual perspective, since the flavor symmetry of the boundary CFT becomes gauge symmetry in the bulk.

See [Bhardwaj, Tachikawa] for the classification of $\mathcal{N}=2$ gauge theories.



Constraints on matter multiplets

Gauge anomaly should be absent:

- Asymptotic freedom (negative beta function) $b_0 = \left(3h^{\vee}\right)$
- (trace-less) symmetric, anti-symmetric.

$$\mathcal{A}(\mathbf{R}_i) = 0$$

$$\bigvee -\sum_{i} T(\mathbf{R}_{i}) \right) \ge 0$$

Above condition restrict the matter representations to fundamental, adjoint,

Let us restrict ourselves to the gauge theories flow to superconformal theories.

Superconformal fixed point

- Necessary condition: Non-anomalous U(1) R-symmetry $\operatorname{Tr} RT^{a}T^{b} = 0 \leftrightarrow T(\operatorname{adj}) + \sum_{i} T(R_{i})(r_{i} - 1) = 0$
- Due to the superconformal symmetry, the conformal anomalies are fixed by the trace anomalies of R-symmetry. [Anselmi, Freedman, Grisaru, Johansen]

$$a = \frac{3}{32} \left(3 \operatorname{Tr} R^3 - \operatorname{Tr} R \right) , \quad c = \frac{1}{32} \left(9 \operatorname{Tr} R^3 - 5 \operatorname{Tr} R \right)$$

- candidate R-symmetries.
- The superconformal R-symmetry is fixed $\frac{\partial a_{\text{trial}}}{\partial a_{\text{trial}}} = 0$ ∂R

• The R-symmetry is not always determined via anomaly constraint. There can be a family of

by 'a-maximization':

$$\frac{\partial^2 a_{\text{trial}}}{\partial R^2} < 0$$

[Intriligator, Wecht]



Decoupling of operators along the RG flow

- Important caveat in a-maximization: accidental symmetry
- Some of the gauge invariant operators may seem to violate the unitarity bound: $\Delta \ge 1$.
- Plausible scenario: such an operator gets decoupled along the RG flow and becomes free with $\Delta_{\mathcal{O}}=1.$ [Kutasov, Parnavhev, Sahakyan]
- One can remove the decoupled free field by introducing a 'flip field' X and a superpotential coupling $W = X \mathcal{O}$. [Barnes, Intriligator, Wecht, Wright]
- Redo the a-maximization until no operator gets decoupled.



Simple large N gauge theory with O(N) degrees of freedom

'Simplest' Large N SCFT

Matter contents:



Gauge invariant operators:

It looks like any other gauge theories with a sparse low-lying spectrum.

[Agarwal, S 1912]

- Coulomb branch operators: Φ^n , $2 \le n \le N$
- dressed mesons: $Q\Phi^n \widetilde{Q}, \ 0 \le n \le N-1$
- 'baryon': $Q(\Phi Q)(\Phi^2 Q) \dots (\Phi^{N-1}Q)$
- 'anti-baryon': $\widetilde{Q}(\Phi \widetilde{Q})(\Phi^2 \widetilde{Q}) \dots (\Phi^{N-1} \widetilde{Q})$

This theory flows to a superconformal fixed point in the IR.



Detailed analysis of the simple model

Compute the trial central charge "a" from the trial R-charge

$$\operatorname{Tr} R_{\Phi}^{1,3} = (N^2 - 1)(1^{1,3} + (R_{\Phi} - 1)^{1,3}) + 2N(1 - NR_{\Phi} - 1)^{1,3})$$

$$\uparrow$$

$$\operatorname{dim}(SU(N))$$

$$\operatorname{dim}(\Box) + \operatorname{dim}(\overline{\Box})$$

Anomaly constraint: $TrRT^{a}T^{b} = 0$ $C_{2}(adj)(1 + (R_{\Phi} - 1)) + 2C_{2}(\Box)(R_{Q} - 1) = 0$ $\uparrow \qquad \uparrow \qquad \uparrow$ R[Gaugino]=1 always $\frac{1}{2}$ R-1 for the fermionic N partner

$$a_{\rm tr}(R_{\Phi}) = \frac{3}{32} \left(3 {\rm Tr} R_{\rm tr}^3 - {\rm Tr} R_{\rm tr} \right)$$



$$a_{\rm tr}(R_{\Phi}) = \frac{3}{32} \left(3 {\rm Tr} R_{\rm tr}^3 - {\rm Tr} R_{\rm tr} \right)$$

Let us maximize "a" with respect to R-charge:



$$\frac{\partial a_{\rm tr}}{\partial R_{\Phi}} = 0 , \quad \frac{\partial^2 a_{\rm tr}}{\partial R_{\Phi}^2} < 0$$

$$\frac{2 + \sqrt{20N^6 - 17N^4 + 1}}{8(2N^4 - N^2 + 1)}$$



Is this okay? We should check consistence!

R-charges of some of the gauge-invariant operators



These unitary violating operators get decoupled along the RG flow.

This results in accidental U(1) symmetry that acts on the decoupled free field. It invalidates the a-maximization procedure.



Let us perform a-maximization again, with additional field X.

$$\frac{\partial a_{\rm tr}}{\partial R_{\Phi}} = 0 , \quad \frac{\partial^2 a_{\rm tr}}{\partial R_{\Phi}^2} < 0$$

To take care of such decoupled operator, introduce a 'flip field' and a F-term

 $W = X \mathrm{Tr} \Phi^2$

Which sets $Tr\Phi^2 = 0$ in the chiral ring. Equivalently, it gives a mass for the "free field" $Tr\Phi^2$.

Upon performing a-maximization again, we obtain:



No unitary violating operator for N=2. More operators decouple for N>2.

$SU(2) + 1 adj + 2 \Box \rightarrow interacting SCFT + 1 free field$

QΦQ $Q \Phi^2 \tilde{Q}$ 2 3

- bound.
- decouple for low *n*.
- None of the 'baryons' decouple. $\Delta_R \sim O(N)$
- The central charges can be computed using the trace anomaly: $a = \frac{3}{32} \left(3 \text{Tr}R^3 \text{Tr}R \right) , \quad c = \frac{1}{32} \left(9 \text{Tr}R^3 5 \text{Tr}R \right)$

- Coulomb branch operators: Φ^n , $2 \le n \le N$
- dressed mesons: $Q\Phi^n \widetilde{Q}, \ 0 \le n \le N-1$
- 'baryon': $Q(\Phi Q)(\Phi^2 Q)\dots(\Phi^{N-1}Q)$
- 'anti-baryon': $\widetilde{Q}(\Phi \widetilde{Q})(\Phi^2 \widetilde{Q}) \dots (\Phi^{N-1} \widetilde{Q})$

• For fixed N, we can repeat this procedure until no operator violates the unitarity

• Some of the Coulomb branch operators $Tr\Phi^n$ and the dressed mesons $Q\Phi^nQ$

Feature 1: The O(N) degrees of freedom



The degrees of freedom grows as $O(N^1)$ instead of the natural matrix degrees of freedom $O(N^2)$! The ratio *a/c* does *not* asymptotes to 1. (Non-holographic)

 $a \simeq 0.500819 N - 0.692539$ $c \simeq 0.503462 N - 0.640935$

 $a/c \sim 0.994757 - 0.111888/N$

Feature 2: Dense spectrum



It does not seem to exhibit confinement/deconfinement transition.

The spectrum of chiral operators form a dense band, instead of being sparse! (analog of the Liouville theory? Decompactification?)

Deformation of SU(N) + 1 adj + $N_f=1$ by flipping

Matter contents:

Superpotential:

Chiral operators:

 $M_i, X_i \ (i = 1, ..., N - 1)$ "Coulomb branch op" $X \equiv Q^N \Phi^{N(N-1)/2}, Y \equiv \tilde{Q}^N \Phi^{N(N-1)/2}, Z \equiv \tilde{Q} \Phi^{N-1} Q$ "Higgs branch op" $\mathcal{M}_H = \mathbb{C}^2 / \mathbb{Z}_N$ $XY = Z^N$

This theory flows to the (A₁, A_{2n-1}) Argyres-Douglas theory, which is a 'non-Lagrangian' $\mathcal{N}=2$ SCFT.

 $\overline{Q} \ ilde{Q} \ ilde{Q} \ \Phi$

[Maruyoshi, JS 1606] [Maruyoshi, JS 1607]

$$a = \frac{12N^2 - 5N - 5}{24(N+1)} , \ c = \frac{3N^2 - N - 5}{6(N+1)}$$

$$\Delta_{M_i} = \frac{2N - i + 1}{N + 1}, \quad \Delta_{X_i} = \frac{3N - N}{N - N}$$

 (M_i, X_i) form an $\mathcal{N}=2$ chiral multiplet (\mathscr{E}).

$$\Delta_X = \Delta_Y = N , \ \Delta_Z = 2$$

$$\frac{\Delta_X^2}{B^2} = 1 < \frac{9C_T}{40C_{V,B}} = \frac{3N^2 - N - 1}{2N^2}$$





The Weak Gravity Conjecture holds.

Dense/O(N) theories behaves similar to the Argyres-Douglas theories! ("N=1 AD theories")

Classification & Weak Gravity Conjecture

Classifying large N theories

- Let us classify all possible supersymmetric large N gauge theories in 4d with the following conditions:
 - The gauge group is simple: G=SU(N), SO(N), Sp(N)
 - The flavor symmetry is fixed as we take large N limit.
 - No superpotential except the flip for the decoupled ops (at the moment).
- In the context of AdS/CFT: flavor symmetry of the boundary CFT = gauge symmetry in the bulk.

Another simple example (no a-maximization) SU(N) with symmetric tensor

- Another simple example: SU(N) with a pair of symmetric tensor
- No need for a-maximization. Rsymmetry fixed by anomaly constraint

$$N + (N+2)(R_S - 1) = 0 \rightarrow R_S = \frac{2}{N+2}$$

• Two types of gauge-invariant ops.

	SU(N)	U(1)s	U(1) _R
S		1	Rs
Ĩ		-1	Rs

Gauge-invariant operators

 $\operatorname{Tr}(S\widetilde{S})^{n} \quad \det S \equiv \epsilon \,\epsilon SS \cdots S$ $\Delta_{\operatorname{Tr}(S\widetilde{S})^{n}} = \frac{3}{2} \cdot 2n \cdot R_{S} = \frac{6n}{N+2}$ $\Delta_{\det S} = \frac{3}{2}N \cdot R_{S} = \frac{3N}{N+2}$



Gauge-invariant operators

$$\Delta_{\det S} = \frac{3}{2}N \cdot R_S = \frac{3N}{N+2}$$

$$a \simeq \frac{95 N^4 + 199 N^3 + 39 N^2 - 164 N - 8}{72 (N+2)^3}$$

$$c \simeq \frac{30 N^4 + 61 N^3 + 15 N^2 - 36 N - 16}{24 (N+2)^3}$$

$$\frac{a}{c} \xrightarrow[N \gg 1]{} \frac{19}{18} \cdot$$

 $\operatorname{Tr}(S\widetilde{S})^n \quad \det S \equiv \epsilon \epsilon SS \cdots S$

$$\Delta_{\mathrm{Tr}(S\widetilde{S})^n} = \frac{3}{2} \cdot 2n \cdot R_S = \frac{6n}{N+2}$$

 $\operatorname{Tr}(SS)^n$ Some low-lying operators decouple $\frac{6n}{N+2} \le 1 \quad \Longrightarrow \quad n \le \left| \frac{N+2}{6} \right|$

Upon removing these decoupled operators (by flipping or simply subtracting), the central charges can be computed to obtain:

$$\begin{array}{cccc} \frac{88}{N \gg 1} & \frac{95}{72}N, \\ & & & \\ N \gg 1 \end{array} \begin{array}{c} \frac{95}{72}N, \\ & & \\ \frac{5}{4}N, \end{array} \end{array} \begin{array}{c} \text{Notice that it has a>c.} \\ & & \\ \text{cf) SQCD: a$$





Theory	β_{matter}	chiral	dense	N_f
$1 \operatorname{\mathbf{Adj}} + N_f (\Box + \overline{\Box})$	$\sim N$	Ν	Y	$N_f \ge 1$
$\Box + 1 \overline{\Box} + N_f (\Box + \overline{\Box})$	$\sim N$	Ν	Y	$N_f \ge 0$
$\square + 1 \square + N_f (\square + \square)$	$\sim N$	Ν	Y	$N_f \ge 4$
$+1\overline{\square} + 8\overline{\square} + N_f (\square + \overline{\square})$	$\sim N$	Y	Y	$N_f \ge 0$
$\Box + 2 \overline{\Box} + N_f (\Box + \overline{\Box})$	$\sim 2 N$	Ν	Ν	$N_f \ge 0$
$\overline{\Box} + 1 \overline{\Box} + 8 \underline{\Box} + N_f (\Box + \overline{\Box})$	$\sim 2 N$	Y	Ν	$N_f \ge 0$
$\overline{\Box} + 1 + 1 + 1 + N_f (\Box + \overline{\Box})$	$\sim 2 N$	Ν	Ν	$N_f \ge 0$
$1 \square + 2 \square + 8 \square + N_f (\square + \square)$	$\sim 2 N$	Y	Ν	$N_f \ge 0$
$+2\square + 16\square + N_f (\square + \square)$	$\sim 2 N$	Y	Ν	$N_f \ge 0$
$+1 \Box + 1 \Box + N_f (\Box + \Box)$	$\sim 2 N$	Ν	Ν	$N_f \ge 0$
$\Box + 2 \Box + N_f (\Box + \Box)$	$\sim 2 N$	Ν	Ν	$N_f \ge 0$
$\Box \Box + 1 \Box + 8 \Box + N_f (\Box + \Box)$	$\sim 2 N$	Y	Ν	$N_f \ge 0$
$+1\square +1\square +N_f (\square + \square)$	$\sim 2 N$	Ν	Ν	$N_f \ge 0$
2 $\operatorname{Adj} + N_f (\Box + \overline{\Box})$	$\sim 2 N$	Ν	Ν	$N_f \ge 0$
$\overline{\Box}) + 2 (\overline{\Box} + \overline{\Box}) + N_f (\Box + \overline{\Box})$	$\sim 3 N$	Ν	Ν	$0 \le N_f \le 2$
$\square + 3 \square + N_f (\square + \square)$	$\sim 3 N$	Ν	Ν	$0 \le N_f \le 6$
$+2\Box +2\overline{\Box} + N_f (\Box + \overline{\Box})$	$\sim 3 N$	Ν	Ν	$0 \le N_f \le 4$
$+1(\Box + \Box) + 1(\Box + \Box)$	$\sim 3 N$	Ν	Ν	•
$+1\square +1\square +N_f (\square + \square)$	$\sim 3 N$	Ν	Ν	$0 \le N_f \le 2$
3 Adj	$\sim 3 N$	Ν	Ν	

Theory	β_{matter}	dense spectru
$1 \Box + N_f \Box$	$\sim N$	Y
$1 \square + N_f \square$	$\sim N$	Y
$2 \Box + N_f \Box$	$\sim 2 N$	Ν
$1 \Box + 1 \Box + N_f \Box$	$\sim 2 N$	Ν
$2\square + N_f \square$	$\sim 2 N$	Ν
	$\sim 3 N$	Ν

Theory	β_{matter}	dense spectrum
$1 \Box + 2N_f \Box$	$\sim N$	Υ
1 \square $+ 2N_f$ \square	$\sim N$	Υ
$2 \Box + 2N_f \Box$	$\sim 2N$	Ν
$1 \Box + 1 \Box + 2N_f \Box$	$\sim 2N$	Ν
$2\square + 2N_f\square$	$\sim 2N$	Ν
$2 \Box + 1 \Box + 2N_f \Box$	$\sim 2N$	Ν
$1 \Box + 2 \Box + 2N_f \Box$	$\sim 2N$	Ν
$3\square + 2N_f \square$	$\sim 3N$	Ν
	$\sim 3N$	Ν



SO(N) theories



Feature 3: Multiple bands eg) SU(N) + 1 adj + $N_f=2$



Figure 6: Plot of a/c vs N for the SU(N) theory with 1 adjoint and $N_f = 2$. The orange curve fits the plot with $a/c \sim 0.936734 - 0.162684/N$.



Figure 8: Dimensions of single-trace gauge-invariant operators including baryons in SU(N)+1 Adj +2 (\Box + $\overline{\Box}$) theory. The baryons(red) form another band above the band of Coulomb branch operators and mesons.

The ratio of central charges a/c does not go to 1.

We see the **secondary band** of size O(N). They are formed by 'baryons'.

- 'baryon': $Q(\Phi Q)(\Phi^2 Q) \dots (\Phi^{N-1}Q)$
- 'anti-baryon': $\widetilde{Q}(\Phi \widetilde{Q})(\Phi^2 \widetilde{Q}) \dots (\Phi^{N-1} \widetilde{Q})$

Supersymmetric analog of 'band' theory?

Sparse vs Dense spectrum



Out of 35 classes of all possible large *N* gauge theories, 8 of them have **dense spectrum** and the rest have sparse spectrum.

> Sparse: The gap is O(1) Dense: The gap is O(1/N)

When do we have a dense spectrum?

- Large N SUSY gauge theory with a dense spectrum is rather common.
- This is the case when the matter representation is 'small'. $\sum T(R_i) \sim h^{\vee} + O(1)$
- This is equivalent to having the 1-loop beta function coefficient to be 'large'. $b_0 = 3h^{\vee} \sum T(R_i)$
- The R-charges (dimensions) for the rank 2 tensor matters scale as 1/N, which also results in $a \sim c \sim O(N)$. Typically a/c does NOT asymptotes to 1.
- The gauge-invariant operators formed out of rank 2 tensors are responsible for the dense spectrum.

Dense spectrum = O(N) degrees of freedom

- The dense model does not satisfy the criterions to have a semiclassical gravity dual. Not a 'Holographic CFT'.
- Nevertheless, AdS/CFT tells us that it is dual to a 'some' quantum gravity in AdS.
 - What kind of bulk theory is it?
 - I do not know.
- But it still satisfies a property that is believed to be true in any (non-AdS) quantum gravity.

Weak Gravity Conjecture in Anti-de Sitter space

- The Weak Gravity Conjecture (WGC) asserts that a light charged particle should exist in any consistent Quantum theory of Gravity.
- It allows the extremal black holes to decay (so that we do not have any absolutely stable remnants).
- In AdS/CFT, it can be translated into the existence of an operator with the ratio of scaling dimension and charges below the ratio of central charges.

$$\frac{\Delta^2}{q^2} \le \frac{9C_T}{40C_V}$$

[Arkani-Hamed, Motl, Nicolis, Vafa]

[Nakayama, Nomura]





Checking the WGC for SU(N) + adj + Nf=1 theory



FIG. 4. Test of the Weak Gravity Conjecture for $U(1)_A$. Red: $9C_T/40C_{V,A}$, Blue: Δ^2/q^2 for the lightest charged operators under $U(1)_A$.

Red: Upper bound from the WGC

 $C_T \sim \mathcal{O}(N),$ $C_{V,A} \sim 2N(R_Q - 1)N^2 + N^2(R_\Phi - 1) \sim \mathcal{O}(N^3)$ $C_{V,B} \sim 2N(R_Q - 1) \sim \mathcal{O}(N).$

The Weak Gravity Conjecture is satisfied!



FIG. 5. Test of the Weak Gravity Conjecture for $U(1)_B$. Red: $9C_T/40C_{V,B}$, Blue: Δ^2/q^2 for the baryon operators charged under $U(1)_B$.

Blue: Lightest charged chiral operator under $U(1)_A$ ('meson' or its flip field) and $U(1)_{B}$ ('baryon')

$$, \qquad \frac{C_T}{C_{V,A}} \sim O\left(\frac{1}{N^2}\right) \qquad \frac{C_T}{C_{V,B}} \sim O\left(1\right)$$



WGC for the multiple U(1)'s

- [Cheung-Remmen]
- Consider an arbitrary charged extremal black hole with charge vector and mass (\vec{Q}, M) that decays via charged particles with charges and masses (\vec{q}_i, m_i) . Then
 - $M > \sum_{i} n_{i} m_{i} , \quad \vec{Q} = \sum_{i} n_{i} \vec{q_{i}}$
- Then the charge to mass ratio can be written as (in appropriate unit)

$$\vec{Z} = \frac{\vec{Q}}{M} = \sum_{i} \frac{n_i \vec{q_i}}{M} =$$

This gives the **convex hull condition**.

It was pointed out that we need a stronger condition when there are multiple U(1)'s.

 $= \sum_{i} \frac{n_i \vec{q_i}}{m_i} \frac{m_i}{M} = \sum_{i} \sigma_i \vec{z_i} \qquad \text{with } \vec{z_i} = \vec{q_i}/m_i \text{ and } \sigma_i = n_i m_i/M$ The extremal black hole satisfies $|\vec{Z}| = 1$ and we have the mass fraction $\sum \sigma_i < 1$.

Checking the WGC for the 'dense' theory



 $SU(N) + 1 adj + N_f = 1$

The WGC satisfied for the most examples we studied.

SU(N) + 1 Sym + N_f=1

Why is the WGC true in a non-holographic theory?

- The argument for the WGC is from a semi-classical reasoning based on black hole in Einstein gravity.
- No obvious reason for this to hold in a generic CFT.
- In $AdS_3 = CFT_2$, modular invariance implies the WGC.
- Is it still the case in higher-dimensional CFT/AdS?
 - A 'counter-example': SQCD in the Veneziano limit $N_c, N_f \rightarrow \infty, N_c/N_f = \alpha$
- [Montero, Shiu, Soler] [Dyer, Fitzpatrick, Xin] [Bae, Lee, JS]

[Nakayama, Nomura]



A counter example for the WGC? $-SU(N) + 1S + 1\overline{AS} + 1\Box + 9\overline{\Box}$



Figure 28: Charge-to-mass ratio of light states dual to the gauge-invariant operators. They Figure 29: Plot of the shortest distance from origin to the boundary surfaces of polyhedron fill a polyhedron with (a) 16 surfaces and 10 vertices for $N \ge 29$, (b) 18 surfaces and 12 in charge-to-dimension space. The chiral ring operators do not satisfy the WGC at $N \leq 12$. vertices for odd $N \leq 29$ and (c) 16 surfaces and 12 vertices for even $N \leq 28$.

We do not find a set of chiral (BPS) operators that satisfy the WGC for small value of N.

Possible interpretations:

- 2. There might be a weak version of the WGC that holds for small N.
- 3. WGC cease to hold at highly quantum & stringy regime.



1. There is an operator that we're missing in the non-BPS sector, which makes the WGC valid.



On the landscape of holographic theories

- delicate balancing.
- Or 'non-Lagrangian'.
 - Class S theories: T_N and their cousins.
 - spectrum, but highly stringy.
- N. eg) (A₁, A_{2N}) vs "rank N H_0 "

Most of the known 'holographic' gauge theories are of quiver type with rather

eg. [Benvenuti-Franco-Hanany-Martelli-Sparks]



[Gaiotto, Maldacena]

• "rank N" F-theory SCFTs (H₀, H₁, H₂, D₄, E₆, E₇, E₈) have sparse low-lying

[Aharony, Fayyazuddin, Maldacena]

• There is an **ambiguity** in choosing a family of theories labelled by an integer

Generic $\mathcal{N}=1$ deformations

- point to obtain a new SCFT.
 - Deform by a relevant operator (R<2) $W = \mathcal{O}$.
 - W = MO.
- Some of the operators can decouple along the RG flow.
- Accidental/non-manifest global symmetry may appear.
- superconformal index)

[Maruyoshi, Nardoni, JS 1806] [Maruyoshi, Nardoni, JS work in progress]

• One can consider all possible $\mathcal{N}=1$ preserving deformation around a superconformal fixed

• Flip a 'super-relevant' operator (R<4/3) by introducing a singlet and a superpotential

• New relevant operators can appear at the end of RG flow. (Dangerously irrelevant operators)

Operators that are **not in the chiral ring** may violate the unitarity bound. (can be checked via



Classification of "small N" superconformal gauge theories

- with fixed matter content by repeating the deformation.
 - SU(2) + adjoint + 1 fundamental flavor: 34 SCFTs
 - SU(3) + adjoint + 1 fundamental flavor: 41 SCFTs
 - SU(2) + adjoint + 2 fundamental flavor: > 400 SCFTs*
 - Sp(2) + adjoint + 1 fundamental flavor: ~ 300 SCFTs*
 - G2 + adjoint + 1 fundamental flavor: > 200 SCFTs*
- Any universality?

[Maruyoshi, Nardoni, S in progress]

One can classify all possible SCFTs that can be obtained from a gauge theory





(incomplete) Tree of RG flows for the SU(2) + adj + 1 fundamental flavor



Figure 5: The distribution of central charges (a, c) for the fixed points realizes as SU(3) gauge theory with one adjoint and one fundamental. There are 41 fixed points in total and 14 of them has no flavor symmetry. The yellow/blue dots represent the ones with/without flavor symmetry.



Figure 7: The distribution of central charges (a, c) for the fixed points realized from deforming the SU(2) gauge theory with one adjoint and 4 fundamentals $(N_f = 2)$. There are 817 candidate fixed points up to level 5.



Summary

- An 'exotic' large N gauge theory: $O(N^1)$ degrees of freedom, dense low-lying spectrum.
- Such large N gauge theories are simple and common. $(\mathcal{N}=1 \text{ analogue of Argyres-Douglas theory})$
- them have dense spectrum.
- CFT since AdS/CFT goes beyond semiclassical Einstein gravity.
- via semiclassical reasoning (up on appropriate correction).

• 35 classes of large N superconformal gauge theories with simple gauge group. 8 of

• One can hope to extract universal features of quantum gravity by studying large N

The WGC maybe a generic feature of quantum gravity even though it is obtained



Future direction

- What is the bulk interpretation of this model with F=O(N)? Unrolling of extra-dimension? Higher-spin gravity?
- Can we prove/disprove a version of WGC in AdS? Any other 'swampland conjectures' to test/formulate?
- Phase structure of the dense models?
- Further classification superpotential, flip-fields, non-simple (quiver) gauge theories.
- Can we find similar results in other dimensions? Especially 3d N=2 gauge theories. AdS₄/CFT₃

Thank you!